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Sustainable Supply Chain Management

Universidade do Minho

Curriculum Development
of Master's Degree Program in
Industrial Engineering for Thailand Sustainable Smart Industry

Inventory Management and Risk Pooling

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Why Is Inventory Important?

- Distribution and inventory (logistics) costs are quite substantial
- Inventory-related decisions have a significant impact on customer service level & supply chain systemwide cost

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Why Is Inventory Required?

- Uncertainty in customer demand
  - Shorter product lifecycles lead to lack of historical data about demand
  - More competing products make it difficult to estimate demand for a specific product
- Uncertainty in supplies
  - Quality/Quantity/Costs/Delivery Times
- Delivery lead times
- Incentives for larger shipments

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Inventory Management-Demand Forecasts

Holding the right amount at the right time is difficult!

- Uncertain demand makes demand forecast critical for inventory related decisions:
  - What to order?
  - When to order?
  - How much is the optimal order quantity?
- Approach includes a set of techniques
  - INVENTORY POLICY!!

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Single Stage Inventory Control

Variety of techniques

- Economic Lot Size Model
- Demand Uncertainty
- Single Period Models
- Initial Inventory
- Multiple Order Opportunities
- Continuous Review Policy
- Variable Lead Times
- Periodic Review Policy
- Service Level Optimization

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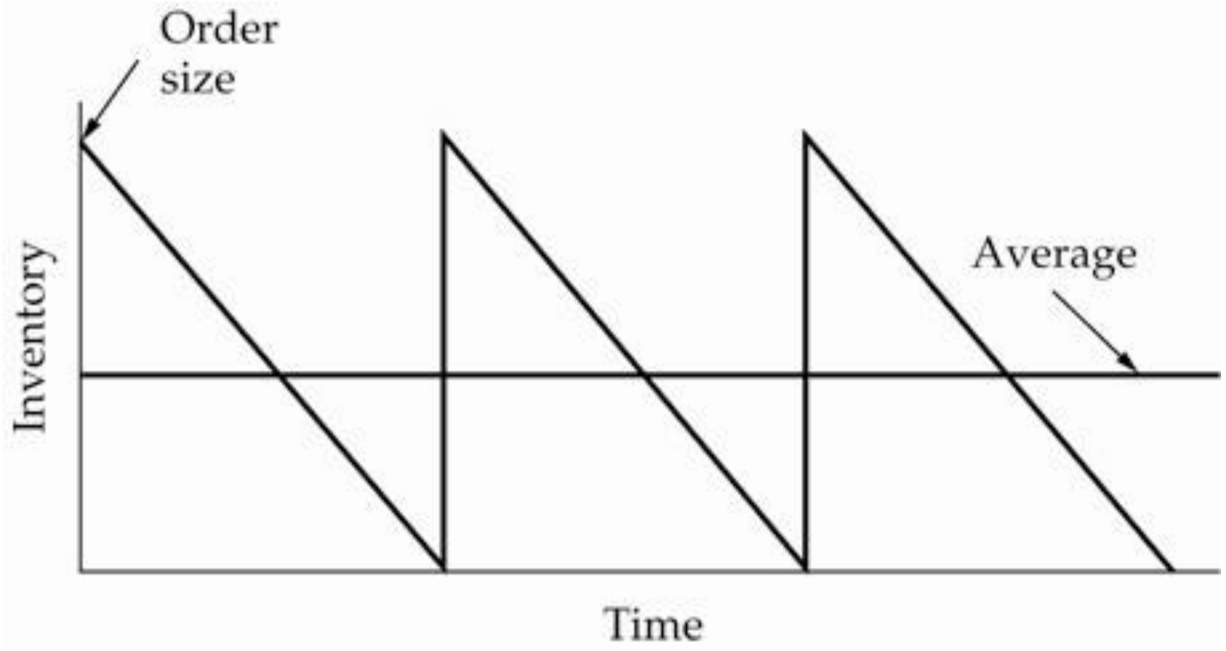
Economic Lot Size Model



Inventory level as a function of time

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Assumptions

- *D* items per day: Constant demand rate
- *Q* items per order: Order quantities are fixed, i.e., each time the warehouse places an order, it is for *Q* items.
- *K,* fixed setup cost, incurred every time the warehouse places an order.
- *h,* inventory carrying cost accrued per unit held in inventory per day that the unit is held (also known as, *holding cost*)
- Lead time = 0

  (the time that elapses between the placement of an order and its receipt)
- Initial inventory = 0
- Planning horizon is long (infinite).

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Deriving EOQ

Total cost at every cycle: $K + \frac{hTQ}{2}$

Average inventory holding cost in a cycle: $\frac{Q}{2}$

Cycle time: $T = \frac{Q}{D}$

Average total cost per unit: $\frac{KD}{Q} + \frac{hQ}{2}$

$$Q^* = \sqrt{\frac{2KD}{h}}$$

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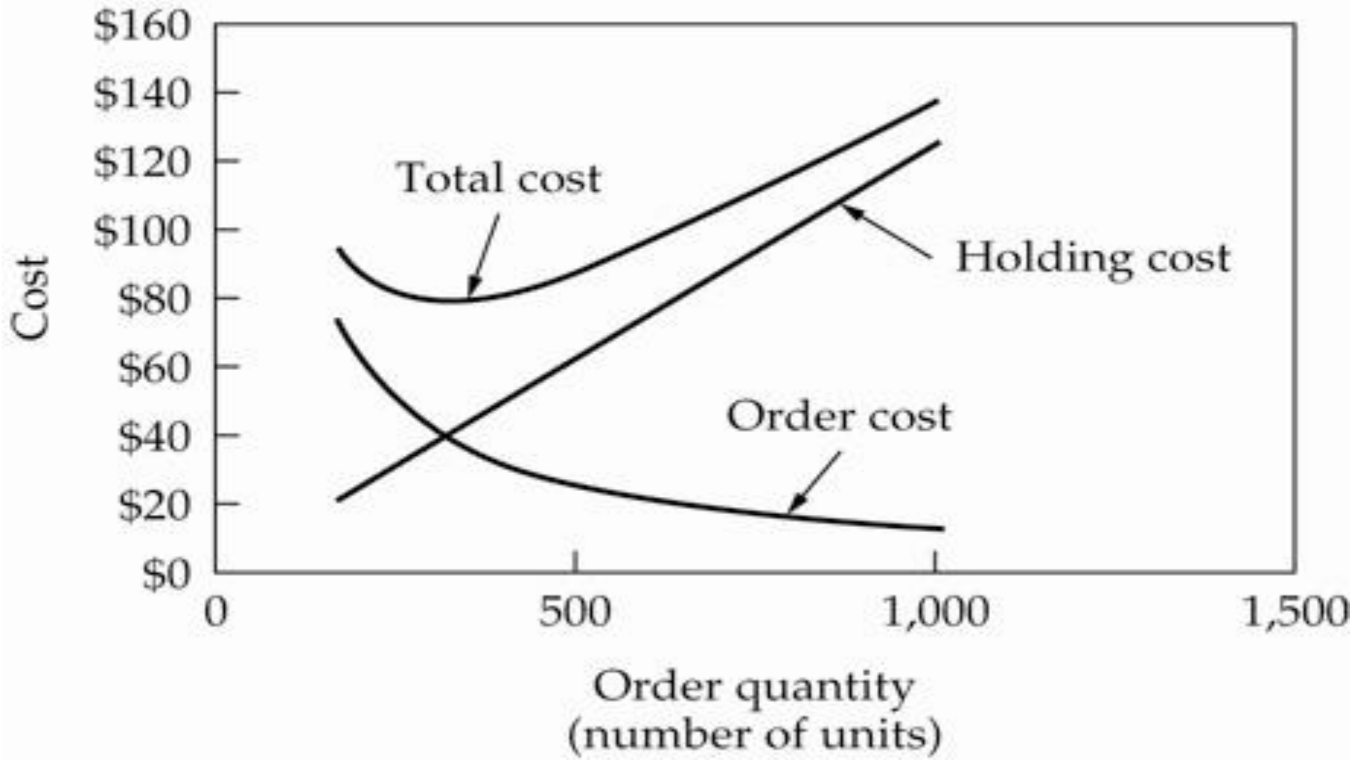
EOQ: Costs



Economic lot size model: total cost per unit time

Note: Total inventory cost is relatively insensitive to order quantities!

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Demand Uncertainty

- The forecast is always wrong
  - It is difficult to match supply and demand
- The longer the forecast horizon, the worse the forecast
  - It is even more difficult if one needs to predict customer demand for a long period of time
- Aggregate forecasts are more accurate.
  - More difficult to predict customer demand for individual SKUs
  - Much easier to predict demand across all SKUs within one product family

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Single Period Models

Short lifecycle products

- One ordering opportunity only
- Order quantity to be decided before demand occurs

  - Order Quantity > Demand => Dispose excess inventory
  - Order Quantity < Demand => Lose sales/profits

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Single Period Models

- The firm can use historical data to
  - identify a variety of demand scenarios
  - determine probability each of these scenarios will occur
- Given a specific inventory policy
  - The firm can determine the profit associated with a particular scenario
  - Hence, given a specific order quantity
    - weight each scenario’s profit by the likelihood that it will occur
    - determine the average, or expected, profit for a particular ordering quantity.
- Order the quantity that maximizes the average profit.

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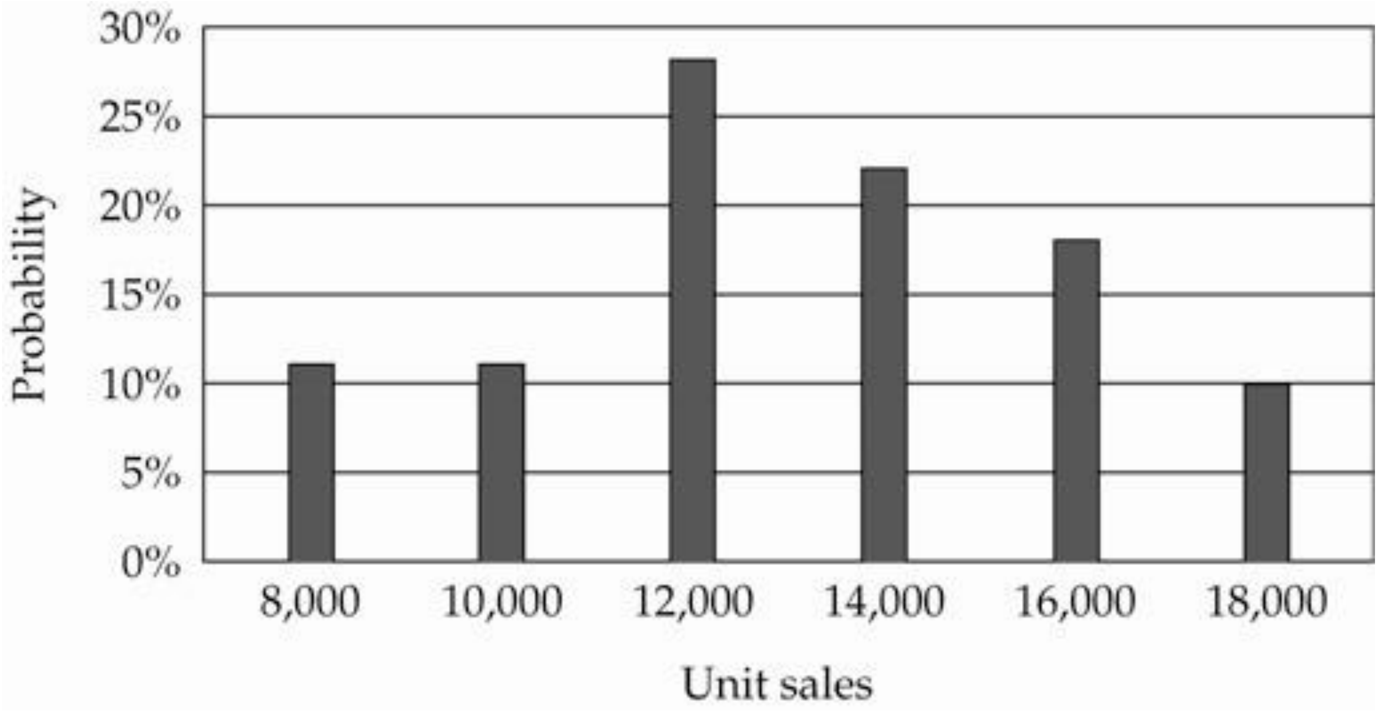
Single Period Model Example



Probabilistic forecast for demand of swimsuit

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Additional Information

- *Fixed production cost:* $100,000
- *Variable production cost per unit:* $80.
- During the summer season, selling price: $125 per unit.
- *Salvage value:* Any swimsuit not sold during the summer season is sold to a discount store for $20.

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Two Scenarios

- Manufacturer produces 10,000 units while demand ends at 12,000 swimsuits

  Profit

  $= 125(10{,}000) - 80(10{,}000) - 100{,}000$

  $= \$350{,}000$

- Manufacturer produces 10,000 units while demand ends at 8,000 swimsuits

  Profit

  $= 125(8{,}000) + 20(2{,}000) - 80(10{,}000) - 100{,}000$

  $= \$140{,}000$

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Probability of Profitability Scenarios with Production = 10,000 Units

- Probability of demand being 8000 units = 11%
  - Probability of profit of $140,000 = 11%
- Probability of demand being 12000 units = 27%
  - Probability of profit of $350,000 = 27%
- Total profit = Weighted average of profit scenarios

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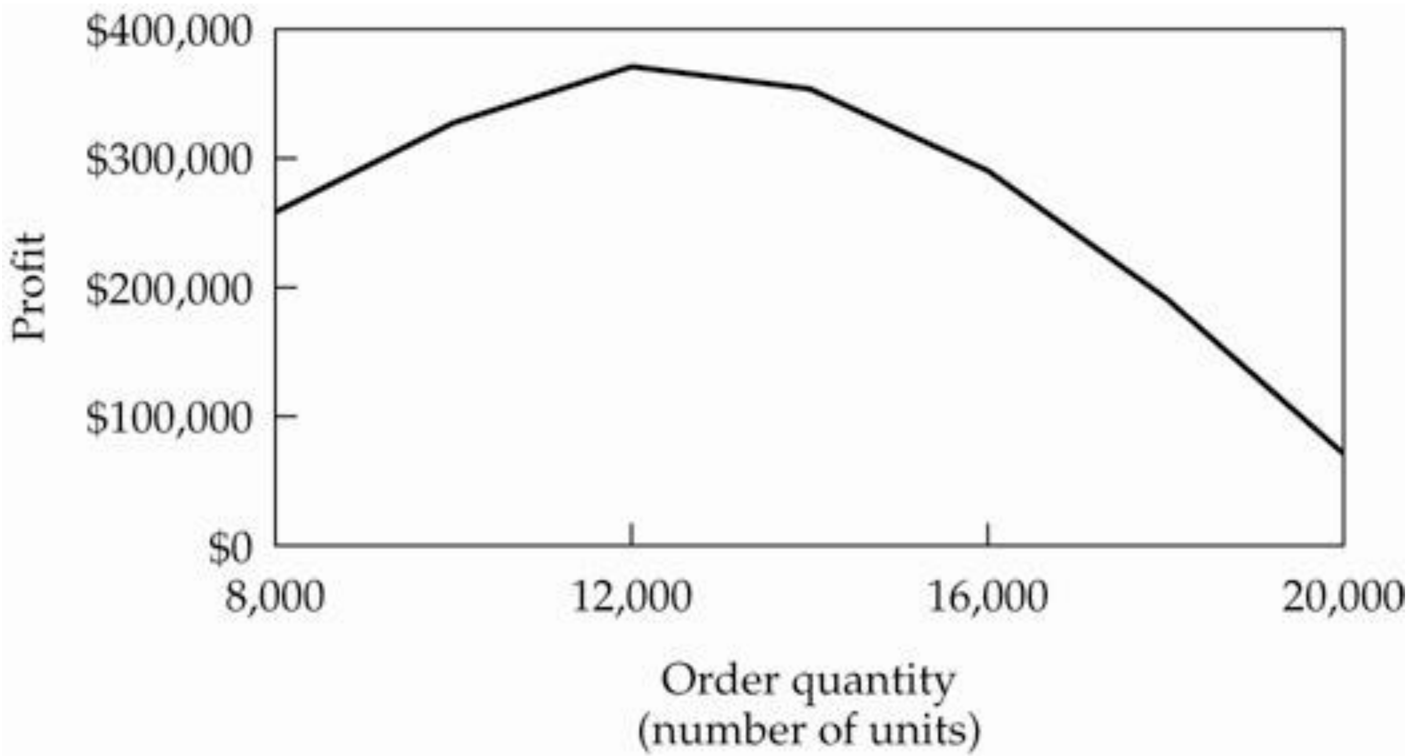
Order Quantity that Maximizes Expected Profit



Average profit as a function of production quantity

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Relationship Between Optimal Quantity and Average Demand

- Compare marginal profit of selling an additional unit and marginal cost of not selling an additional unit

- Marginal profit/unit =

  Selling Price - Variable Ordering (or, Production) Cost

- Marginal cost/unit =

  Variable Ordering (or, Production) Cost - Salvage Value

<u>The rules are</u>:

- If Marginal Profit > Marginal Cost => Optimal Quantity > Average Demand
- If Marginal Profit < Marginal Cost => Optimal Quantity < Average Demand

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For the Swimsuit Example

- Average demand = 13,000 units.
- Optimal production quantity = 12,000 units.

- Marginal profit = $45
- Marginal cost = $60.

- Thus, Marginal Cost > Marginal Profit
  => optimal production quantity < average demand.

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Risk-Reward Tradeoffs

- Optimal production quantity maximizes average profit is about 12,000 ($370,700)
- Producing 9,000 units or producing 16,000 units will lead to about the same average profit of $294,000.
- If we had to choose between producing 9,000 units and 16,000 units, which one should we choose?

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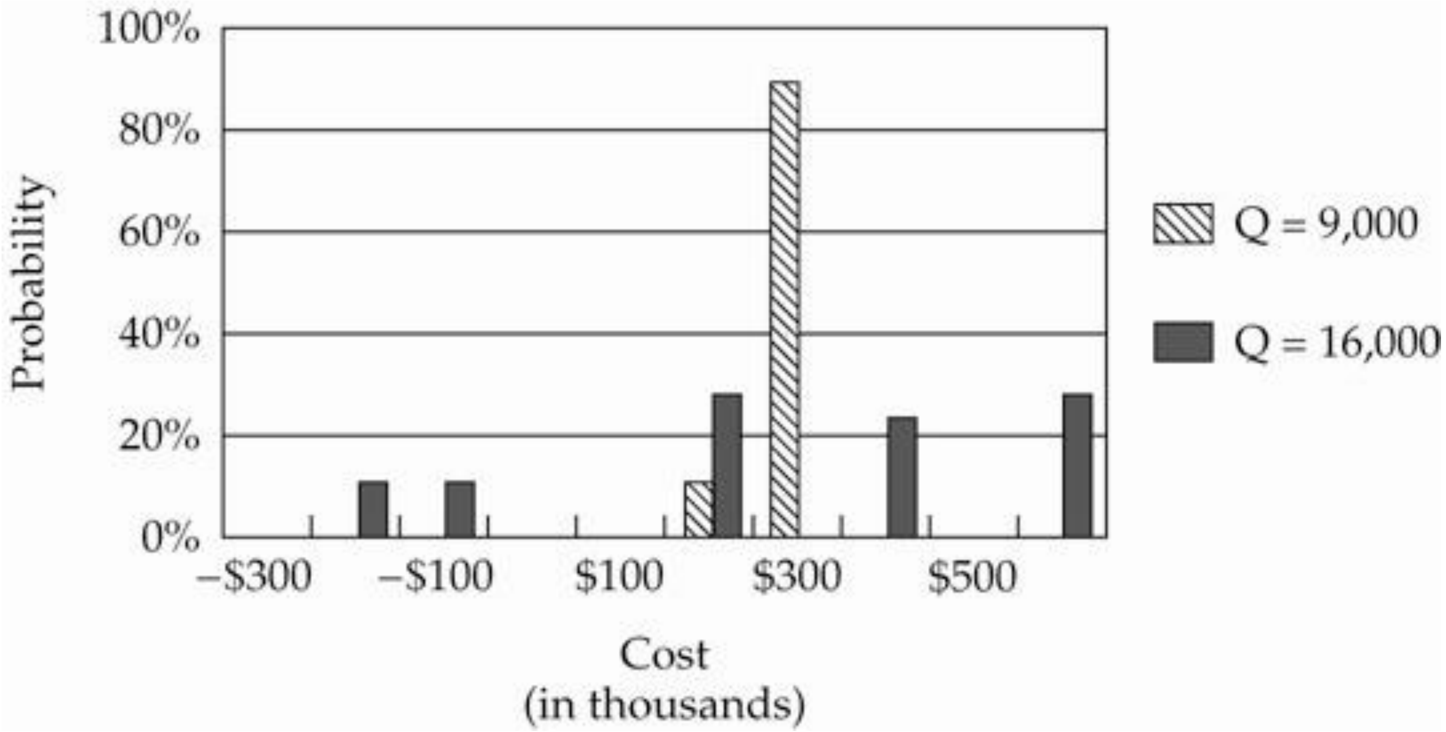
Risk-Reward Tradeoffs



A frequency histogram of profit

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Risk-Reward Tradeoffs

- Production Quantity = 9000 units
  - Profit is:
    - either $200,000 with probability of about 11 %
    - or $305,000 with probability of about 89 %
- Production quantity = 16,000 units.
  - Distribution of profit is not symmetrical.
  - Losses of $220,000 about 11% of the time
  - Profits of at least $410,000 about 50% of the time
- With the same average profit, increasing the production quantity:
  - Increases the possible *risk* (i.e., larger loss)
  - Increases the possible *reward* (i.e., larger gain)

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Observations

- The optimal order quantity is not necessarily equal to forecast, or average demand.
- As the order quantity increases, average profit typically increases until the production quantity reaches a certain value, after which the average profit starts decreasing.
- <u>Risk/Reward trade-off</u>: As we increase the production quantity, <u>both</u> risk and reward increases.

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What If the Manufacturer has an Initial Inventory?

- Trade-off between:
  - Using on-hand inventory to meet demand and avoid paying fixed production cost: need sufficient inventory stock
  - Paying the fixed cost of production and not have as much inventory

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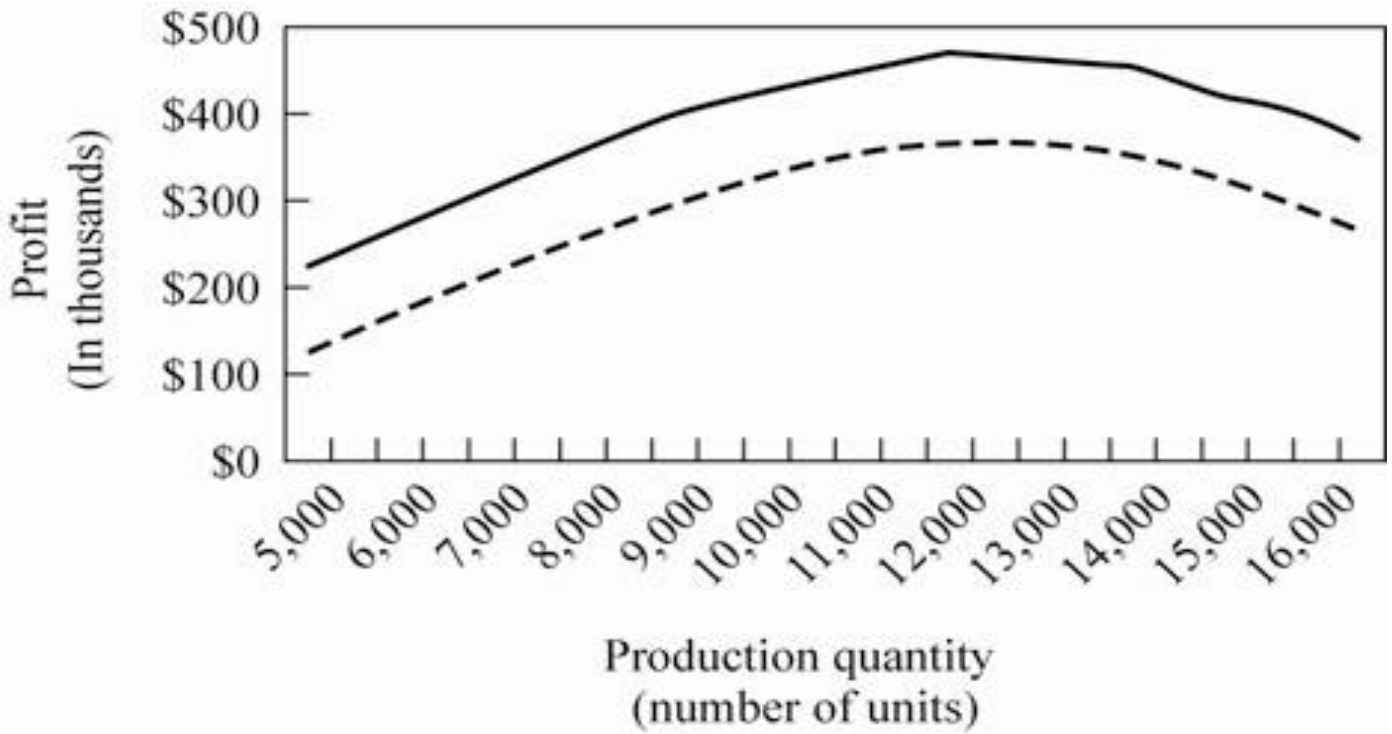
Initial Inventory Solution



Profit and the impact of initial inventory

The dotted curve: expected profit; The solid curve: exclude the fixed prod. cost

The solid curve is above the dotted curve an amount equals to fixed prod. cost

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Manufacturer Initial Inventory = 5,000

- If nothing is produced
  - Average profit =

  225,000 (from the solid curve) + 5,000 x 80 = 625,000
- If the manufacturer decides to produce
  - Production should increase inventory from 5,000 units to 12,000 units.
  - Average profit =

  370,700 (from the dotted curve) + 5,000 x 80 = 770,700

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Manufacturer Initial Inventory = 10,000

- No need to produce anything because
  - average profit > profit achieved if we produce to increase inventory to 12,000 units

- If we produce, the most we can make on average is a (real) profit of $370,700.
  - Same average profit with initial inventory of 8,500 units and not producing anything.
  - If initial inventory < 8,500 units => produce to raise the inventory level to 12,000 units.
  - If initial inventory is at least 8,500 units, we should not produce anything

(*s*, *S*) policy or (*min, max*) policy

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Multiple Order Opportunities

In many practical situations, products might be order repeatedly

REASONS TO HOLD INVENTORY

- **To balance annual inventory holding costs and annual fixed order costs.**
- **To satisfy demand occurring during lead time.**
- **To protect against uncertainty in demand.**

TWO POLICIES

- **Continuous review policy**
  - inventory is reviewed continuously
  - an order is placed when the inventory reaches a particular level or reorder point.
  - inventory can be continuously reviewed (computerized inventory systems are used)

- **Periodic review policy**
  - inventory is reviewed at regular intervals
  - appropriate quantity is ordered after each review.
  - it is impossible or inconvenient to frequently review inventory and place orders if necessary.

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Continuous Review Policy

- Daily demand is random and follows a normal distribution.
- Every time the distributor places an order from the manufacturer, the distributor pays a fixed cost, *K*, plus an amount proportional to the quantity ordered.
- Inventory holding cost is charged per item per unit time.
- Inventory level is continuously reviewed, and if an order is placed, the order arrives after the appropriate lead time.
- If a customer order arrives when there is no inventory on hand to fill the order (i.e., when the distributor is stocked out), the order is lost.
- The distributor specifies a required *service level*.

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Continuous Review Policy

- *AVG* = Average daily demand faced by the distributor
- *STD* = Standard deviation of daily demand faced by the distributor
- *L* = Replenishment lead time from the supplier to the distributor in days
- *h* = Cost of holding one unit of the product for one day at the distributor
- α = service level. This implies that the probability of stocking *out is 1 -* α

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Continuous Review Policy

- *(Q,R)* policy – whenever inventory level falls to a reorder level *R*, place an order for *Q* units

- What is the value of R?

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Continuous Review Policy

Average demand during lead time: $L \times AVG$

Safety stock: $z \times STD \times \sqrt{L}$

Reorder level, *R*: $L \times AVG + z \times STD \times \sqrt{L}$

Order quantity, *Q*: $Q = \sqrt{\dfrac{2K \times AVG}{h}}$

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Service Level & Safety Factor, *z*

Service Level	90%	91%	92%	93%	94%	95%	96%	97%	98%	99%	99.9%
z	1.29	1.34	1.41	1.48	1.56	1.65	1.75	1.88	2.05	2.33	3.08

z is chosen from statistical tables to ensure
that the probability of stockouts during lead time is exactly 1 - *α*

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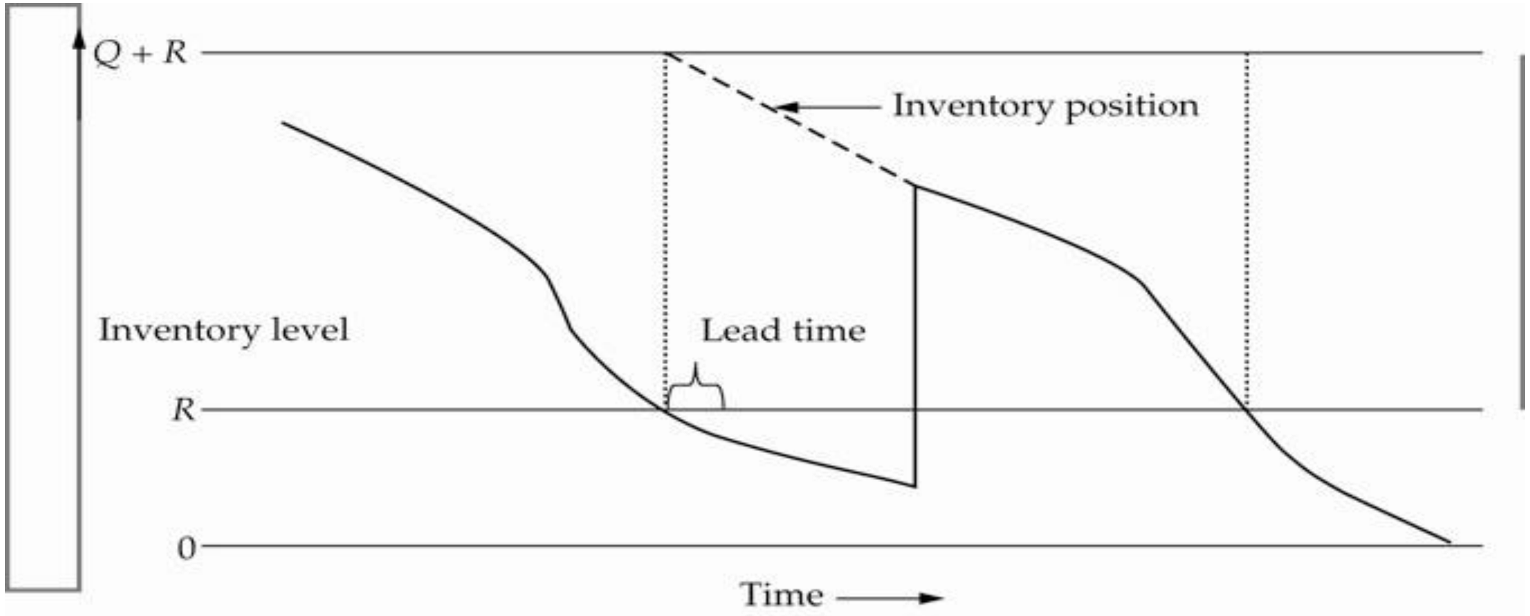
Inventory Level Over Time

Inventory level as a function of time in a (*Q*,*R*) policy



Inventory level before receiving an order = $z \times STD \times \sqrt{L}$

Inventory level after receiving an order = $Q + z \times STD \times \sqrt{L}$

Average Inventory = $\frac{Q}{2} + z \times STD \times \sqrt{L}$

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Continuous Review Policy Example

- A distributor of TV sets that orders from a manufacturer and sells to retailers
- Fixed ordering cost = $4,500
- Cost of a TV set to the distributor = $250
- Annual inventory holding cost = 18% of product cost
- Replenishment lead time = 2 weeks
- Expected service level = 97%

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Continuous Review Policy Example

Month	Sept	Oct	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug
Sales	200	152	100	221	287	176	151	198	246	309	98	156

Average monthly demand = 191.17
Standard deviation of monthly demand = 66.53

Average weekly demand = Average Monthly Demand/4.3
Standard deviation of weekly demand = Monthly standard deviation/sqrt(4.3)

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Continuous Review Policy Example

Parameter	***Average weekly demand***	***Standard deviation of weekly demand***	***Average demand during lead time***	***Safety stock***	***Reorder point***
Value	***44.58***	***32.08***	***89.16***	***86.20***	***176***

Weekly holding cost = $\frac{0.18 \times 250}{52} = 0.87$

Optimal order quantity = $Q = \sqrt{\frac{2 \times 4{,}500 \times 44.58}{.87}} = 679$

Average inventory level = *679/2* + *86.20* = *426*

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Periodic Review Policy

- Inventory level is reviewed periodically at regular intervals
- An appropriate quantity is ordered after each review
- Two Cases:
  - Short Intervals (e.g. Daily)
    - Define two inventory levels s and S
    - During each inventory review, if the inventory position falls below s, order enough to raise the inventory position to S.
    - ***(s, S) policy***
  - Longer Intervals (e.g. Weekly or Monthly)
    - May make sense to always order after an inventory level review.
    - Determine a target inventory level, the base-stock level
    - During each review period, the inventory position is reviewed
    - Order enough to raise the inventory position to the base-stock level.
    - ***Base-stock level policy***

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(s,S) policy

- Calculate the Q and R values as if this were a continuous review model
- Set s equal to R
- Set S equal to $R+Q$.

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Base-Stock Level Policy

- Determine a target inventory level, the base-stock level
- Each review period, the inventory position is reviewed and order enough to raise the inventory position to the base-stock level
- Assume:

  r = length of the review period

  L = lead time

  AVG = average daily demand

  STD = standard deviation of this daily demand.

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Base-Stock Level Policy

- Average demand during an interval of $r + L$ days= $(r+L)\times AVG$
- Safety Stock= $z \times STD \times \sqrt{r+L}$

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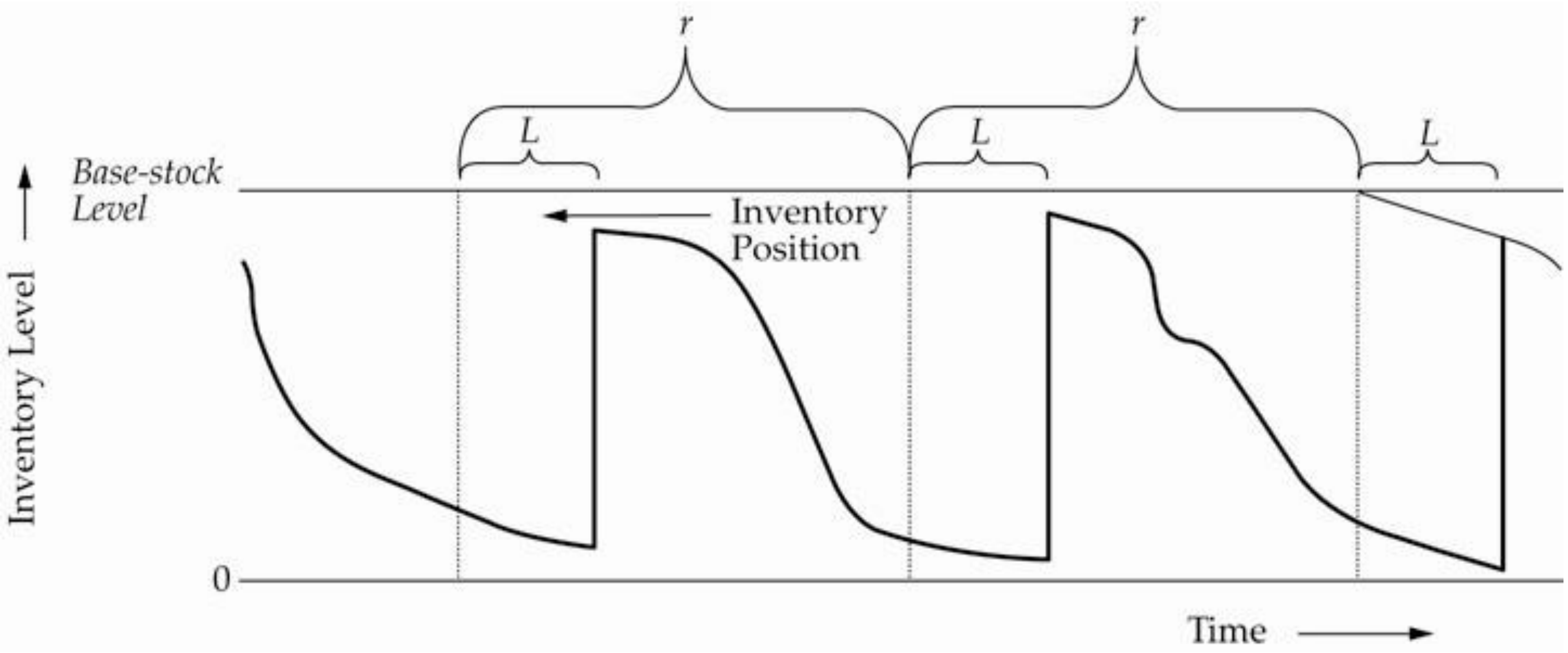
Base-Stock Level Policy



Inventory level as a function of time in a periodic review policy

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Base-Stock Level Policy Example

- Assume:
  - distributor places an order for TVs every 3 weeks
  - Lead time is 2 weeks
  - Base-stock level needs to cover 5 weeks
- Average demand = 44.58 x 5 = 222.9
- Safety stock = $1.9 \times 32.8 \times \sqrt{5}$
- Base-stock level = 223 + 136 = 359
- Average inventory level = $\frac{3 \times 44.58}{2} + 1.9 \times 32.08 \times \sqrt{5} = 203.17$
- Distributor keeps 5 (= 203.17/44.58) weeks of supply.

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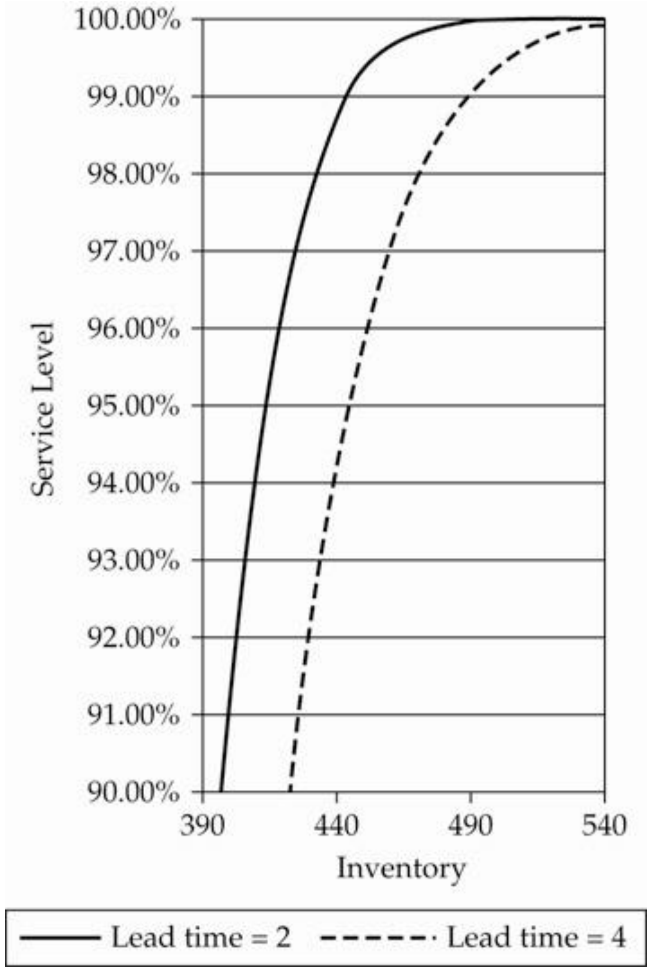
Service Level Optimization



Service level versus inventory level as a function of lead time

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Trade-Offs

- Everything else being equal:
  - the higher the service level, the higher the inventory level.
  - for the same inventory level, the longer the lead time to the facility, the lower the level of service provided by the facility.
  - the lower the inventory level, the higher the impact of a unit of inventory on service level and hence on expected profit

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Risk Pooling

- Demand variability is reduced if one aggregates demand across locations.

- More likely that high demand from one customer will be offset by low demand from another.

- Reduction in variability allows a decrease in safety stock and therefore reduces average inventory.

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Demand Variation

- Standard deviation measures how much demand tends to vary around the average
  - Gives an absolute measure of the variability

- Coefficient of variation is the ratio of standard deviation to average demand
  - Gives a relative measure of the variability, relative to the average demand

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ACME Risk Pooling Case

- Electronic equipment manufacturer and distributor
- 2 warehouses for distribution in Massachusetts and New Jersey (partitioning the northeast market into two regions)
- Customers (that is, retailers) receiving items from warehouses (each retailer is assigned a warehouse)
- Warehouses receive material from Chicago
- Current rule: 97 % service level
- Each warehouse operate to satisfy 97 % of demand (3 % probability of stock-out)

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New Idea

- Replace the 2 warehouses with a single warehouse (located some suitable place) and try to implement the same service level 97 %
- Delivery lead times may increase
- But may decrease total inventory investment considerably.

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Historical Data

PRODUCT A								
Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Massachusetts	33	45	37	38	55	30	18	58
New Jersey	46	35	41	40	26	48	18	55
Total	**79**	**80**	**78**	**78**	**81**	**78**	**36**	**113**
PRODUCT B								
Week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Massachusetts	0	3	3	0	0	1	3	0
New Jersey	2	4	3	0	3	1	0	0
Total	**2**	**6**	**3**	**0**	**3**	**2**	**3**	**0**

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Summary of Historical Data

Statistics	Product	Average Demand	Standard Deviation of Demand	Coefficient of Variation
Massachusetts	**A**	39.3	13.2	0.34
Massachusetts	**B**	1.125	1.36	1.21
New Jersey	**A**	38.6	12.0	0.31
New Jersey	**B**	1.25	1.58	1.26
Total	**A**	77.9	20.71	0.27
Total	**B**	2.375	1.9	0.81

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Inventory Levels

	Product	Average Demand During Lead Time	Safety Stock	Reorder Point	Q
Massachusetts	**A**	39.3	25.08	65	132
Massachusetts	**B**	1.125	2.58	4	25
New Jersey	**A**	38.6	22.8	62	131
New Jersey	**B**	1.25	3	5	24
Total	**A**	77.9	39.35	118	186
Total	**B**	2.375	3.61	6	33

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Savings in Inventory

- Average inventory for Product A:
  - At NJ warehouse is about 88 units (=SS + Q/2)
  - At MA warehouse is about 91 units
  - In the centralized warehouse is about 132 units
  - Average inventory reduced by about 36 percent
- Average inventory for Product B:
  - At NJ warehouse is about 15 units
  - At MA warehouse is about 15 units
  - In the centralized warehouse is about 20 units
  - Average inventory reduced by about 43 percent

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Critical Points

- **The higher the coefficient of variation, the greater the benefit from risk pooling**
  - The higher the variability, the higher the safety stocks kept by the warehouses. The variability of the demand aggregated by the single warehouse is lower
- **The benefits from risk pooling depend on the behavior of the demand from one market relative to demand from another**
  - risk pooling benefits are higher in situations where demands observed at warehouses are negatively correlated
- **Reallocation of items from one market to another easily accomplished in centralized systems. Not possible to do in decentralized systems where they serve different markets**

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Centralized vs. Decentralized Systems

- *Safety stock:* lower with centralization
- *Service level:* higher service level for the same inventory investment with centralization
- *Overhead costs:* higher in decentralized system
- *Customer lead time:* response times lower in the decentralized system
- *Transportation costs:* not clear. Consider outbound and inbound costs.

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Managing Inventory in the Supply Chain

- Inventory decisions are given by a single decision maker whose objective is to minimize the system-wide cost
- The decision maker has access to inventory information at each of the retailers and at the warehouse
- Echelons and echelon inventory
  - Echelon inventory at any stage or level of the system equals the inventory on hand at the echelon, plus all *downstream inventory* (downstream means closer to the customer)

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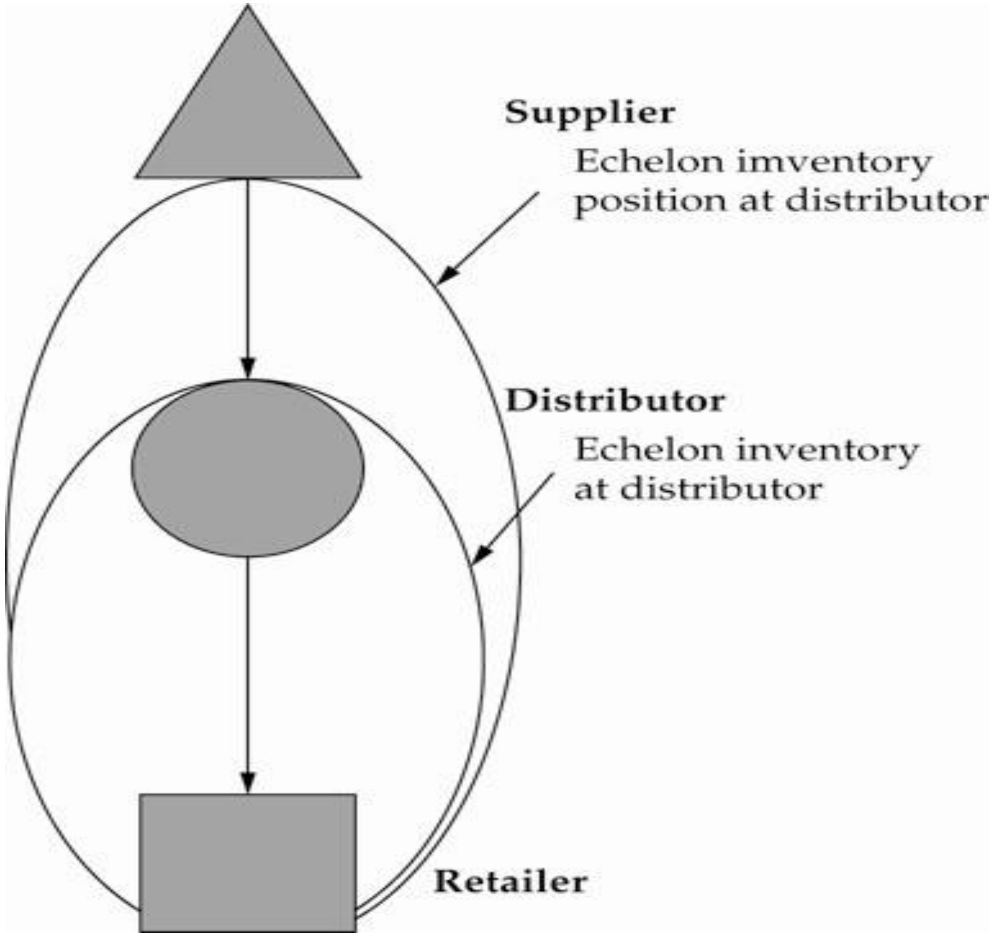
Echelon Inventory



A serial supply chain

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Reorder Point with Echelon Inventory

- L^e = *echelon lead time*,
  - lead time between the retailer and the distributor *plus* the lead time between the distributor and its supplier, the wholesaler.
- *AVG* = average demand at the retailer
- *STD* = standard deviation of demand at the retailer
- Reorder point

$$R = L^e \times AVG + z \times STD \times \sqrt{L^e}$$

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4-Stage Supply Chain Example

- Average weekly demand faced by the retailer is 45
- Standard deviation of demand is 32
- At each stage, management is attempting to maintain a service level of 97% (z=1.88)
- Lead time between each of the stages, and between the manufacturer and its suppliers is 1 week

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Costs and Order Quantities

	K	D	h	Q
retailer	250	45	1.2	137
distributor	200	45	.9	141
wholesaler	205	45	.8	152
manufacturer	500	45	.7	255

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Reorder Points at Each Stage

- For the retailer, R=1*45+1.88*32*$\sqrt{1}$ = 105
- For the distributor, R=2*45+1.88*32*$\sqrt{2}$ = 175
- For the wholesaler, R=3*45+1.88*32*$\sqrt{3}$ = 239
- For the manufacturer, R=4*45+1.88*32*$\sqrt{4}$ = 300

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More than One Facility at Each Stage

- Follow the same approach
- Echelon inventory at the warehouse is the inventory at the warehouse, plus all of the inventory in transit to and in stock at each of the retailers.
- Similarly, the echelon inventory position at the warehouse is the echelon inventory at the warehouse, plus those items ordered by the warehouse that have not yet arrived minus all items that are backordered.

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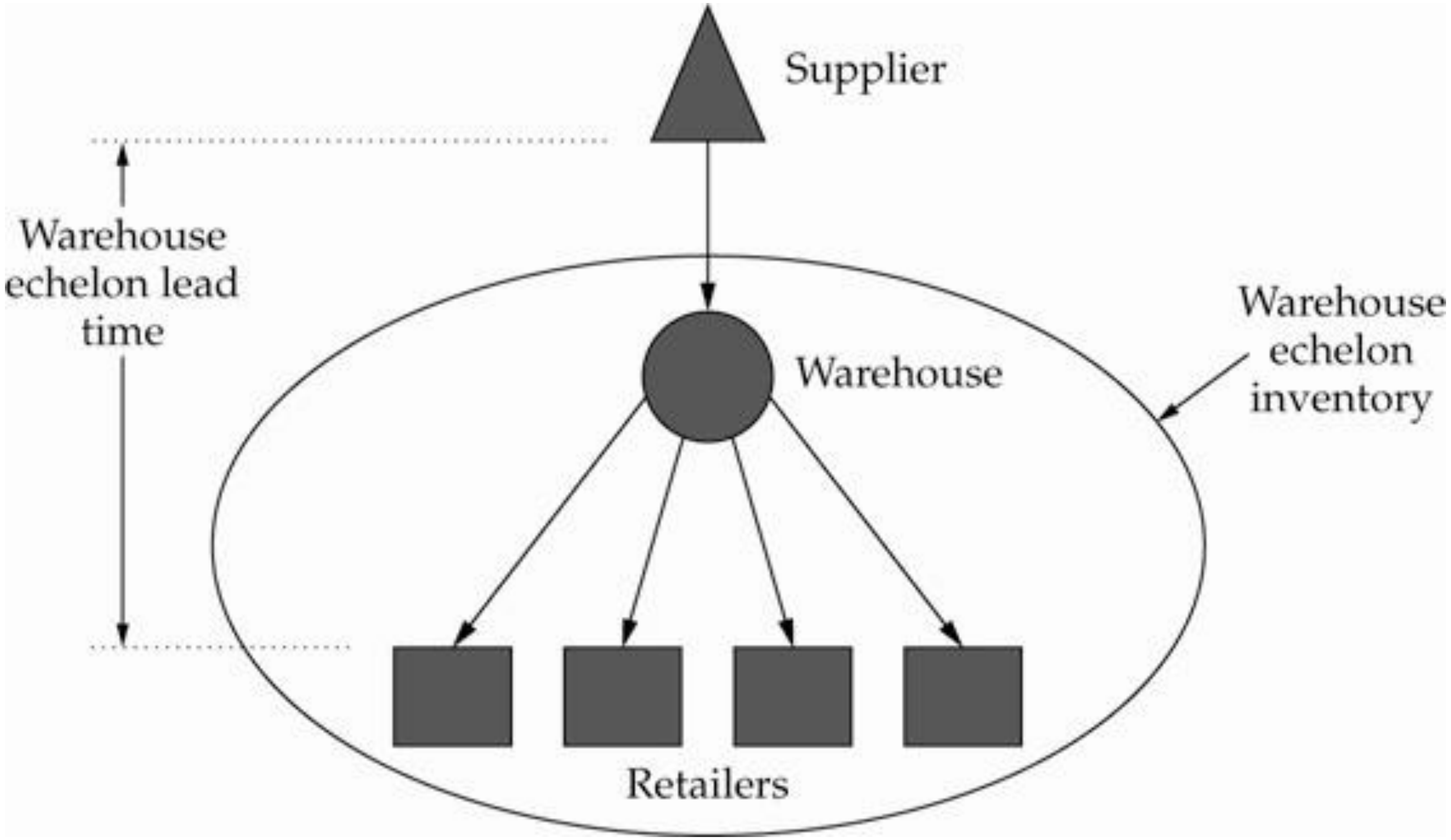
Warehouse Echelon Inventory



The warehouse echelon inventory

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Practical Issues

Top seven inventory reduction strategies

- **Periodic inventory review.**
- **Tight management of usage rates, lead times, and safety stock.**
- **Reduce safety stock levels.**
- **Introduce or enhance cycle counting practice.**
- **ABC approach.**
- **Shift more inventory or inventory ownership to suppliers.**
- **Quantitative approaches.**

FOCUS: not reducing costs but reducing inventory levels. Significant effort in industry to increase *inventory turnover*

$$Inventory\_Turnover\_Ratio = \frac{Annual\_Sales}{Average\_Inventory\_Level}$$

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